Computers Math. Applic. Vol. 17, No. 4–6, pp. 485–503, 1989
Printed in Great Britain.

0097-4943/89 $3.00 + 0.00

MIND'S EYE

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Abstract—The concept of symmetry is one of the great universal principles used to comprehend the enormous amounts of data encountered in both the worlds of natural phenomenon and of abstract knowledge. With the advent of computers, methodology has evolved to process and generate huge amounts of information. This information is often inconsistent and ambiguous and is similar to that encountered by human perception. This article develops some commonalities between applications of symmetry and applications of computer methodology to visual perception (robotic vision), to explore the impact of developing technology on general understandings about human knowledge. These commonalities suggest that advances in robotic vision will enlarge the study of symmetry, reveal astonishing new types of symmetry, and produce unexpected applications of philosophical interrelationships between abstract and perceptual knowledge.

INTRODUCTION

With this second book on symmetry, the editors confirm their belief that the symmetry concept is a basic principle that is useful to explain relationships between aspects of mathematics and physical, biological, and other natural phenomenon [1]. It is a concept whose *superimposition* on computational analysis is one approach for organizing vast amounts of information and computational techniques to produce understandable, accessible and useful results [2].

As these volumes illustrate, symmetry has relevance and meaning for an awe-inspiring range of human intellectual endeavor, ranging from the broad canvas of the arts to the unseeable realm of high energy physics. It has long been associated with beauty in both of the two cultures considered in C. P. Snow's discussions about disparities between the sciences and the arts. It is also discovered with astonishing variety in natural phenomenon and abstract knowledge. The scope of symmetry produces the hope that nature may possess an order that is accessible to the comprehension of the human mind [2].

Yet the meaning of symmetry is not precisely defined. Intuitive understanding of symmetry, derived from visual perceptions of simple geometric forms, appears straightforward. Developing understanding and applications most often proceed by formation of mathematical relations describing a transformation process. This process, in general, represents an implied action on spatial orientation to produce predictable perceptions, often that of an image indistinguishable from the original visual scene. Yet these two volumes illustrate that the symmetry concept encompasses far broader areas of human understanding than those involved with geometric transformation or abstract mathematical formulation. One of the many books dedicated to explaining and exploring the symmetry concept introduces a particular abstract concept of symmetry that is based on linguistic analogy. This clearly extends the study of symmetry to include the far reaches of poetic license [3]. The same book includes a description of symmetry as a disease. The numerous varieties of symmetry invoked in these current volumes imply that a precise definition cannot encompass the full meaning of the symmetry concept and that, in fact, it may not be possible to provide well-defined limitations.

This, then, results in the contention that a fundamental ambiguity is inherent in the concept of symmetry. One purpose of this article is to explore some of the specific ambiguity that arises in applications of the symmetry concept, even in supposedly well-defined areas. The importance of the presence of this ambiguity lies in its association with the nature of visual perception. This article will show, to some extent, that the attempts to develop robotic vision have also encountered

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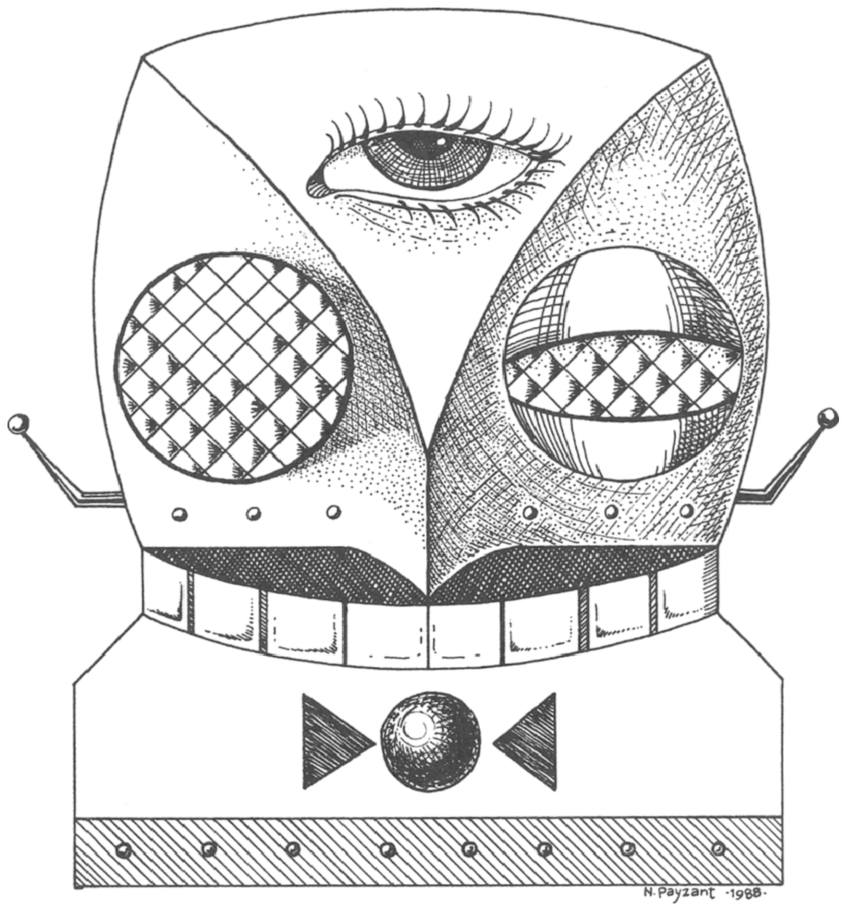


ambiguity as a fundamental part of the attempts to simulate visual perception. This provides an essential conceptual commonality: ideas about symmetry and ideas about machine vision share the much-discussed limitations of human knowledge. They are both subject to the poorly understood, nonrational mental processing involved in creating the perceptions with which our reasoning and thinking begin.

Having established some of these inherent difficulties in defining and applying the symmetry concept and in applying computational technology to creating machine vision, this article suggests that methodologies being developed to deal with these difficulties have a similar basis and might well stimulate new approaches to both areas of research. In conclusion, there will be discussion of how the new computational technology will have a broad impact—possibly in areas of understanding which today appear quite remote from such machine-oriented reasoning.

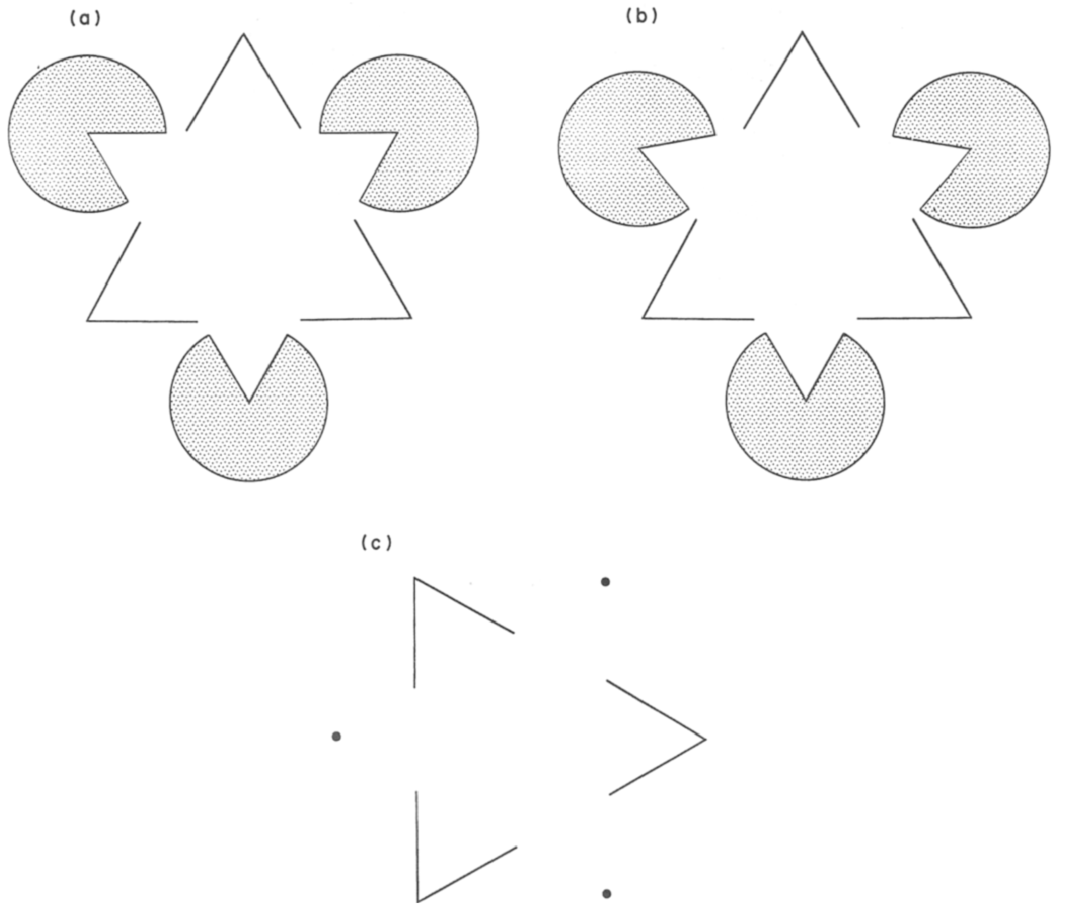
VISION AS PROCESS: GENERAL CONSIDERATIONS

What does it mean to see? It can be argued that seeing is designed to know *what is where*. In this light, vision is the process of discovering what is present in the world and where is its relative location by means of some mental analysis of the stimulus present in a given visual scene. This description of vision as an *information-processing task* is an important starting place, both for the purposes of this article and for the myriad of activity prompted by the idea that computing technology has made possible the creation of a machine that could see. As will be discussed, the exploration of this information-processing task has provided profound insights into the nature of human perception and response to visual scenes with general philosophical implications about human knowledge and understanding. It is of interest here, as an introduction into some the many problems of robotic vision, to present some information about a particular class of human perception, namely that of optical illusion, and the visual information which prompts this type of perception. The purpose of this discussion is to illuminate some of the essential difficulties in creating a simple description of perception in terms of an elementary-information-processing task.

Machine vision has excited research since the early 1950s. In his general review of the developments in this technology, Ladd [4] presents the early realization that certain distinguishing features in a visual scene might be the fundamental elements for construction of visual perception. Edges, shading, texture and many of the other apparent components of visual information were quickly seized upon as the basis for attempts at machine algorithms for reproducing sight. It can be argued that this generally has been the basis for the development of image processing as a separate technical discipline. Certainly much effort using this type of visual classification continues today. However, it was soon realized that there were inherent limitations embodied in this approach to vision, most notably the inability to produce computational schemes that were applicable to generalized visual processing. Given an elaborate computer code and specialized scenes, some apparent discrimination was possible among, for example, shapes in a world of blocks or bagels in a bin [4]. However, such visual analysis was inadequate to reproduce general perception and, in fact, was itself subject to incompleteness and lack of definition. In the physical world, edges are fuzzy, colors or shades ill-defined and irregular, and textures nonuniform. Far more important was the realization that such analysis did not encompass much of the information necessary for perception and that the human mind could construct coherent visualizations almost independent of this type of visual information.

As an illustration of the above ideas, several types of optical illusions will be presented as a background for the discussion on vision and symmetry that follows. These exemplify the extent of the complexity of human perception that is being illuminated by research into robotic vision. Consider the subjective contours discussed by Kanizsa [5]. These are vivid illustrations of the unconscious visual processing from which human perception evolves. Figures 1(a)–(c) give the illusion of triangular figures whose outlines are clearly perceived but are not explicitly present in the visual information of the diagrams. The first figure illustrates a white triangle superimposed over another triangle in a planer arrangement; the white color of this triangle appears brighter than the background although they are in fact identical. The second figure suggests a curved arc and perhaps a three-dimensional arrangement. This effect was achieved merely by rotating the two top *PAC* Man-like figures 10° in a symmetrical fashion and slightly rearranging their location to align the perceived contour generated from the figure below. There are no curved lines involved in generating the illusion of the arc and, in fact, Fig. 1(c) shows that even the triangular cutouts from the circles are not necessary to create a bright undrawn figure.

In an article on the interpretation of visual illusions, Hoffman [6] discusses some of the factors which are likely to be incorporated in this visual processing. It is a fact that the eye is capable of perceiving a spherical shape given only the visual information generated by randomly placed lights located on a rotating sphere in a dark environment. In analyzing this phenomenon, Hoffman emphasizes the use of the learned laws of projection and the observation of rigid, generally smooth objects existing in the physical world. He cites a proof by Shimon Ullman that these two accepted beliefs can, in principle, provide a unique and correct solution to this problem. Hoffman discusses rules regarding perception of curvature in describing the intersection of objects. Hoffman and Richards proposed a principle of Transversality suggesting that the intersection of two surfaces is perceived as a concave discontinuity. They suggest that this provides some understanding about the famous goblet/face illusion of Rubin (Fig. 2). Here the illusion depends on the interpretation of object and background. When the object is considered to be a white goblet intersection with a black background, most of the curves are concave with normals of minimum curvature pointing into the black background. When the black faces are perceived as the objects, the concave intersections with the white background become the recognizable features such as nose, lips and chin whose normals of minimum curvature project into the white background. With respect to symmetry, this phenomenon is strikingly apparent in Fig. 3 (taken from Kanizsa [7]). The intersection of two figures is seen as junctions intruding into each figure as a background rather than the juxtaposition of two symmetric objects. Hoffman concludes that vision is an active inferential process exploiting regularities in the visual world and that mathematical investigation of this inferential power is a promising direction towards greater understanding of human vision.



Figs 1(a)–(c). Kanizsa subjective contour.

The premises and implementation of these ideas are discussed extensively in this article; the illustrations and discussion of this section hopefully provide an introduction into the nature of the problems under consideration. It is noteworthy that these authors share common concerns about inferential, nonconscious visual processing. They emphasize the search for regularities and

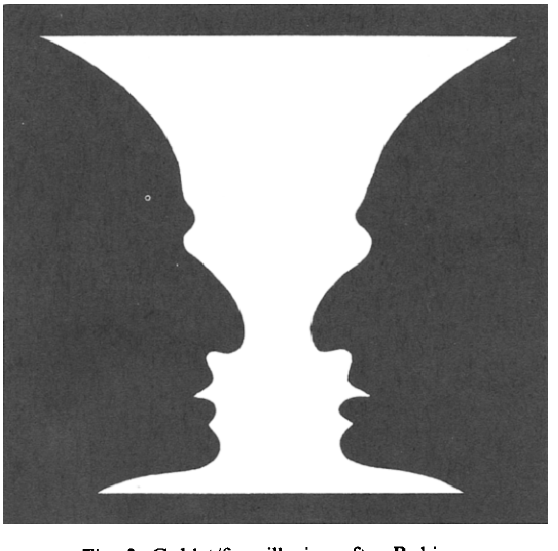


Fig. 2. Goblet/face illusion after Rubin.

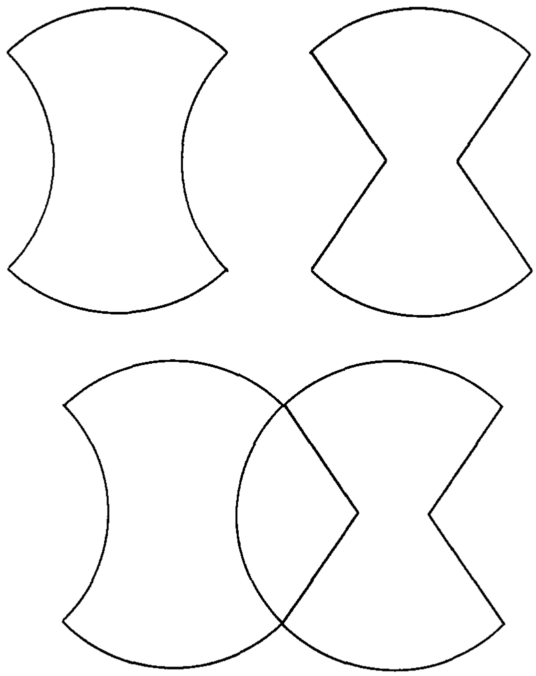


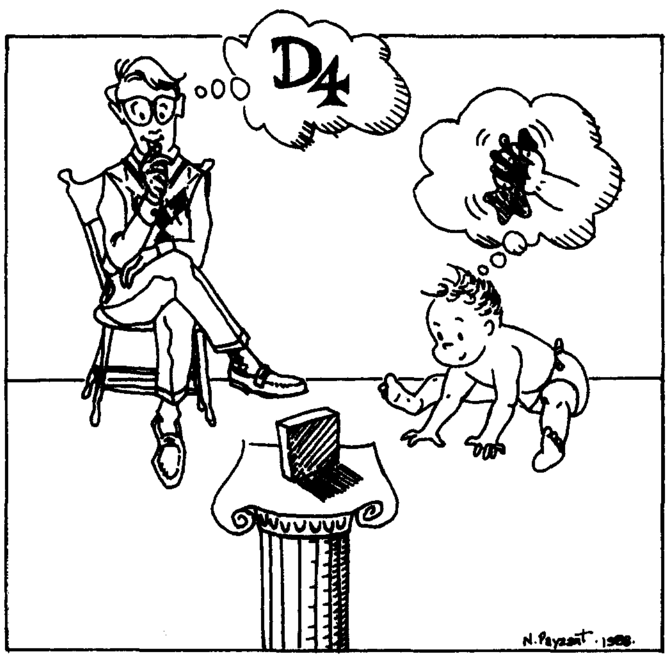
Fig. 3. Kanizsa figure showing perceptual transformation upon juxtaposition of two symmetric objects.

mathematical information, above and beyond that information present in the visual scene, for useful understanding of visual perception.

SYMMETRY: SOME GENERAL OBSERVATIONS

At first glance, symmetry is deceptively simple. First introductions to the symmetry concept often center on elementary transformations of planer figures. Rotations, reflections, inversions and the like become the first examples because it is easy to visualize the manner in which they recreate the original image. The transition to mathematics for concise descriptions then becomes obvious and even satisfying as some of the more elegant symmetries are developed. The mind often finds beauty in observations that are not readily accessible at first thought or apparent without some body of information in which they are embedded. The importance of this line of inquiry, directed towards developing sophisticated tools and language for succinct description of complex physical phenomena, is well illustrated by numerous essays in the first volume of this series [1]. For this article, it is useful to emphasize that inherent in this line of inquiry is a generalized concept of symmetry as a transformation *process*, and depends on mathematical reasoning and logical deduction to develop the language and descriptions for a scientific understanding of phenomenon in the physical world.

The dominance of this approach masks ambiguity about the role symmetry plays in human understanding and knowledge. A pointed example may be derived from considering a square cut from a child's modeling clay. Asked about transformations of this square, the response of a student trained in symmetry is predictable; the elementary symmetry operations are obvious and easily described by equations on paper. For a five-year-old child however, a similar question is likely to result in the square being squeezed into a three-dimensional lump and then rolled again into a square shape. Such a symmetry transformation is not mentioned in text books and is hard to describe by equations. Another example, more directly related to the purpose of this essay, is the presumed symmetry created by visual perception; "... a strong tendency exists in thought to the extent that man may discover nature to be dominated by laws of symmetry even if, in many cases, one suspects he discovers what he himself has put there" [7]. Low-resolution visual information tends to create symmetric perceptions despite marked distortions at higher resolution [8]. The



purpose of these observations is to relate questions about symmetry to generalized questions about human knowledge and understanding.

One of the historical triumphs of the development of science has been the escape from a man-centered universe. Beginning with the Copernician revolution, developments up to the early twentieth century produced a series of laws and rules governing an orderly, predictable universe which appeared to operate in a manner quite independent of the investigator and the experiment. The presumption was clearly almost that of the clockwork natural world whose mechanisms, given enough time and attention, would be revealed and accessible to the human mind in a manner that would allow deterministic predictions of future actions and events. This view still has practical usefulness and seems to be the basis for Einstein's widely quoted awe at the amount of the workings of the physical world that are understandable to human intelligence. With the developments in this century, we have been forced to return, in part, to a mind-centered base for knowledge and understanding. The well-known limits on predictability and powers of observation and on the deterministic view of the physical world have created a new relationship between scientific problems, methods of experiment or analysis, and the nature of the solutions obtained. In a sense, this has created some new perspectives on the nature of scientific investigation and knowledge as exemplified by lines of inquiry relating the practice of physics to the disciplines of oriental religion [9].

In the context of this changing flux of ideas, this article attempts to relate the understanding of symmetry to these more generalized problems of human understanding. The efforts directed towards applying technology to human perception, robotic vision in this instance, have created methodologies that are clearly related to philosophical examinations of the nature of scientific thought. Some observations drawn from the work of Bertrand Russell provide a background for the further discussion of machine vision and the relationship between the symmetry concept and these generalized questions about scientific understanding. The following ideas are brief extractions from the book by Bertrand Russell entitled *Human Knowledge, Its Scope and Limitations* [10].

Russell's work deals with examination of the nature in which the human mind processes information. He posits certain first principles: of relevance to this discussion is the concept of a belief for which no further reason can be given, i.e. a belief that is a postulate based on a certain type of faith about the nature of the physical world and about the mechanisms of human understanding. In a scientific context, an example of such a belief is the sense that perception of a given experimental result repeated over a period of time is indicative of an observation relevant to the description of some portion of the physical world. This example is not completely representative of the complexity of the question posed by Russell since it is the very nature of perception and understanding that is being examined. However, this example raises the basic question of interpretation which is the main concept of importance for the current discussion. Russell suggests, for example, that such a type of scientific belief or principle is interpreted in the least questionable form. This interpretation itself is an example of a premise which, consciously or unconsciously is *assumed* in the reasonings of science. He also suggests that such questions about interpretation imply essential ambiguity when he posits that there are many statements about which we are more certain of their truth than of their meaning.

An important quality of these principles or beliefs, at least in ordinary thinking, is that they are the cause of other beliefs or concepts and are not derivative. This active character is important because these beliefs affect the process of reasoning or deduction. In the scientific context, Russell chooses to emphasize that these beliefs constitute the conscious or unconscious methodology of investigation and reasoning and are *assumed* rather than derived. He is trying to focus on assumptions that have created the great body of scientific knowledge and information. From a historical sense, it is obvious that many revolutions in science have resulted from challenges to the assumptions that governed the interpretation of observations and information in the physical world. It is this relation between assumptions and interpretations which concerns Russell when he says [10, p. 224]:

> "The question of interpretation has been unduly neglected. So long as we remain in the region of mathematical formulation, everything appears precise, but when we seek to interpret them, it turns out that the precision is partially illusionary. Until this matter has been cleared up, we cannot tell with any exactitude what any given science is asserting."

Interpretation then becomes a pivotal word that is central to the ideas of this article. The practical applications of the symmetry concept (both in the abstract and in the physical worlds) involve selective observations, assumptions about the process of transformations, and implicit conditions imposed by the nature of the solution or application desired.

The simple example of the model clay square shows this but the idea deserves further elaboration. It is easily visualized that interpretation of symmetry depends on the time scale of observations in much the same manner as it depends on spatial resolution. The human visual response time that makes television and films such convincing illusions provides a simple example. Time resolved pictures of the familiar rotating pinwheel reveal distinct symmetries that are not observable when it is spinning in the wind. In the extreme, a nonsymmetrical object in a static reference frame relative to the observer may acquire circular symmetry if rotated sufficiently rapidly about any arbitrary axis. Well-known examples of time-frame symmetry appear in experimental science. Molecular structure contains a classic example of phosphorous pentafluoride. When examined in a magnetic resonance experiment, the bond lengths are observed as equal while in an electron diffraction experiment distinct types of bond lengths are observed. The difference arises from the different duration of measurement; the electron diffraction experiment achieves essentially snapshots of molecules that are in static positions compared to the time scale of molecular vibrations and, despite the inherent averaging, resolution of different average bond lengths is possible. The magnetic resonance experiment is ten orders of magnitude slower than the electron diffraction measurements. Thus this experiment measures bond lengths for the molecule throughout the entire time span of a complex vibrational transformation known as psuedorotation. In this transformation, atoms exchange relative orientation and rearrangement of the bonding structure and the lengths are then all observed to be equal.

An important point of this discussion is that symmetry is in fact ambiguous; determination of symmetry depends on factors beyond the inherent properties being examined. Another observation, related to perception of symmetry but also serving as a general comment on the nature of visual perception, is that well-known factors of cultural conditioning influence the observations derived from visual information. Western culture, at least since the fifteenth century, has accepted perspective drawing as representative of three-dimensional (3-D) solid objects or depths of scene. However, there is considerable evidence that this perception of depth in 2-D representations is a learned experience. Dergowski [11] has reviewed experiments designed to test the nature of 3-D perception of culturally remote people. Specifically, with regards to an observation of symmetry, individuals not accustomed to depth perspective built model constructions designed to represent a 3-D perspective sketch of a cube as two squares with diagonal connections on a planer surface as opposed to creating a 3-D construct. It was similarly observed that groups of people from similar cultural background who apparently exhibited 3-D perception suffered confusion when trying to model the optical illusion of an "impossible" Penrose trident. In Fig. 4 the familiar 3-D view of the partial cube seems to have the correct perspective but it becomes flat for many people when rotated 45°; the extended drawing is the representation given by people not accustomed to perspective interpretation of pictures. It is apparent that perception of symmetry in a visual scene is thus ambiguous in this culturally dependant context. The 2-D interpretation is quite different from the 3-D understanding, and each interpretation is clearly a function of factors not present in the visual information at hand.

ROBOTIC VISION: ANALOGIES AND ANALYSIS

Because it appears possible to create machine vision, enormous human effort has been unleashed that encompasses the whole panoply of human knowledge, understanding and technology. It would be tempting to say that this development is being driven by technological advances. Indeed the unique expansion of computer power and information storage has certainly created a radically new set of tools with which to investigate this awesome concept. However, it is more generally true that these technological developments have primarily stimulated the continuing effort to understand human vision as a part of the understanding of human knowledge; a human activity that has existed at least since the time of recorded history (see author's note at end of article). The inherent involvement of these most general lines of inquiry is of particular significance because it demonstrates that advances in technology will influence the common understanding of these ancient problems. It is clear that this technology can provide challenges to the basic assumptions upon which our understanding is founded.

To create a manageable discussion, this section presents robotic vision in the perspective of analytical representations of human perception. It also discusses, in some small detail, the reasoned approach presented by Marr [12] of a single facet of this process, that portion of 3-D vision that creates illusions of depth from seemingly random patterns. As often occurs in scientific investigation, pathological cases (in this situation an optical illusion) allow isolation of specific phenomenon for insight into the understanding of the more general process. There is vigorous debate about the technical developments in this field; it is a broad and fertile research area for many

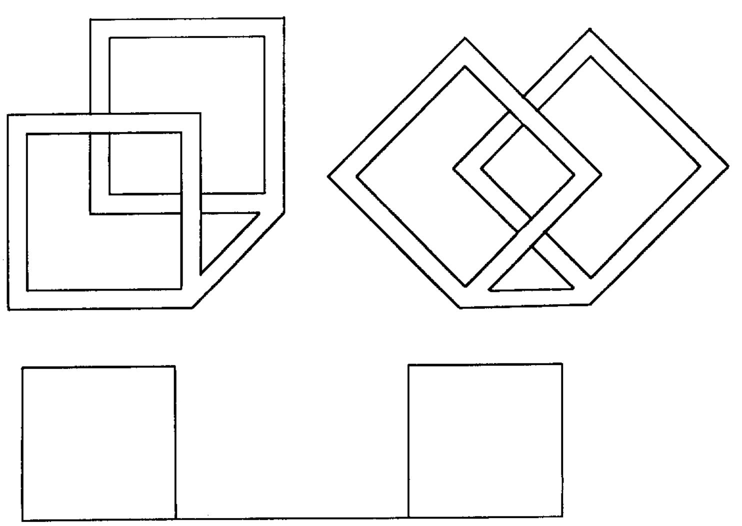


Fig. 4. Figures illustrating cultural relativity of depth perception. Top two representation of cube-like objects show change of depth perception upon orientation. Bottom extended figure is representation of 3-D construct made by people accustomed to 2-D perception.

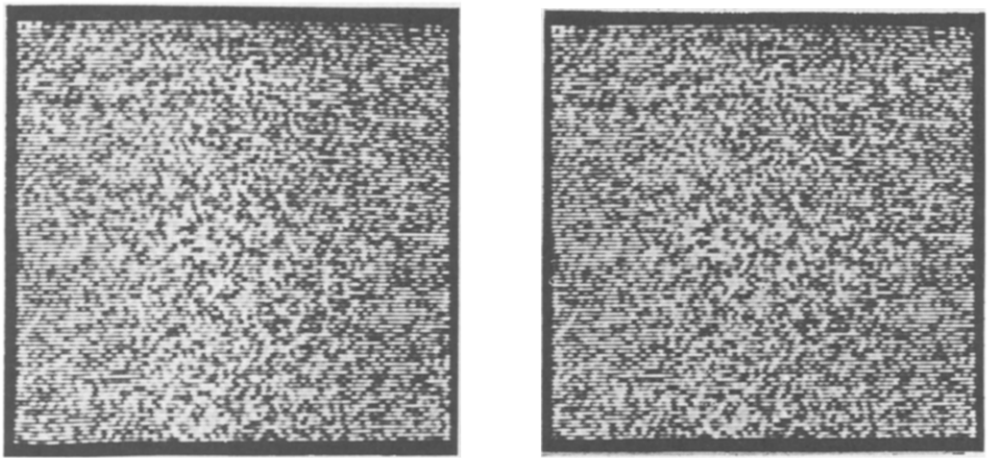


Fig. 5. Julesz stereogram illusion: left and right diagrams differ only by a uniform translation of a square section of random dot image in one image. From "Binocular depth perception of computer-generated patterns" by Bela Julesz from *The Bell System Technical Journal* **XXXIX**(5), 1129 (1960). Published by the American Telephone and Telegraph Company, Copyright 1960, used by permission.

ideas and methodologies [4]. The examples from the work of Marr are chosen as vivid examples of conceptual approaches to this subject, thus providing useful material for this discussion.

A brief and preliminary introduction of the specific example for the basis of our discussion is helpful in creating the context for the more extensive questions of interest. Consider the random dot stereogram experiment of Julesz [13] which is shown in Fig. 5. This consists of a pair of random dot images, identical except that a central square section of one has been uniformly translated a distance away from its original location. Perceived individually, these diagrams have the appearance of completely random images; unless exaggerated, translation of this segment of dots produces little noticeable change. Viewed stereoscopically, the junction of the two images produces the perception of this central square floating above the untranslated background image. The perceived image is a controllable function of the translation process. Depth perception can be related to the concept of stereoscopic fusion through the measure of the various amounts of displacement. Translation processes other than uniform displacment can create different effects on depth perception, and elaborate experiments are also possible with multiple correspondence. Detailed discussion of such extensive research is beyond the purpose of this article, but it is significant to realize the computational technology is creating new laboratories for examining age-old questions of vision and perception.

This phenomenon has been critically analyzed by Marr using a computational approach as a conceptual basis for machines that would create a similar interpretation from such images. It is important to stress that this discussion already raises a fundamental question about robotic vision: that of purpose. This section expands the question of purpose that is the guiding force behind the overall strategic development of robotic vision and is, in part, the driving force behind the development of the technology for accomplishing this end. According to Marr, the purpose of robotic vision is to build a machine that knows *what is where by looking*, that is to build a machine that could process images and discover what is present in the world and where it is located. It is important to realize, however, that such a machine must be able to create a computationally accessible and processable representation of this visual image in order to generate machine preception and in order to make this information useful in the general manner to which human beings are accustomed. This definition of functionality is particularly more elaborate than that for a television camera which can transmit only visual information, albeit transformed, distorted or "enhanced". Marr quotes Austin [14] emphasizing that viable robot vision should also at least roughly correspond to what an ordinary person knows to be true at first hand.

One reason for this emphasis is that visual processing may serve different needs or functions. This is shown by the analysis of the vision of a housefly presented by Reichardt and Poggio [15]. It is clear that about 60% of the fly's visual system is oriented toward processing a few simple relative motion parameters. For example, if the visual field of an object rapidly expands, say from a nearby looming surface, the motor mechanisms for landing are activated. The fly lands in the center of this visual field (inverting if the field is above) and when the feet touch the surface, the power system for the wings is switched off. Another feature of the system is apparently tuned to objects possessing both a small angular dimension in the field of view and a relative motion with respect to the background. This information is apparently delivered in such a manner that a fly a few feet away will be intercepted while an elephant at 100 yd does not register. The investigations propose a simple differential equation that apparently describes directional control. These studies clearly show that the purpose of vision in the fly is primarily to control motion by regulating the connections between the muscle systems upon which survival depends. It is not apparent that more complex information is an important aspect of the fly's visual processing. Lack of more complicated visual processing may well contribute to the speed of the fly's muscular reaction to visual stimulus, about 21 ms.

The purpose of human visual perception is obviously much more complex. It is this complexity that has generated the enormous range of investigations into human psychology and the generalized process known as perception as well as the studies concerning the suitable construct of logical circuitry and computational design that will reproduce in a manageable way the information about *what is where.*

It is important to recognize that Marr begins with the premise that vision is a computationally accessible process. This is not a trivial assumption. This assumption is an acknowledgement of visual perception as an intrinsic product of an unconscious, inferential process. This justifies a logical foundation for a methodological analysis and is patently necessary for any consideration of creating machine vision. From this premise, Marr proposes three levels of approach to dividing the problem into tractable forms: that of computational theory, that of developing representations and algorithms for expressing this theory, and that of hardware implementation for the physical and computational processing of the visual information. He makes a telling case that this is a necessary unified methodology for beginning to deal with the global problems mentioned above. The role of computational theory as the first step in this process cannot be underestimated. In the first place, it begins to formulate, in an explicit way, the nature of the information needed to produce visual perception from external optical stimulus. Secondly, this emphasis on theory provides a mechanism for analyzing and integrating the contraints on visual perception imposed by the active phenomenon of the real world, and for understanding how these constraints may be related to the mental processes of our imagination and thinking.

Computational theory is a broad term. Marr's belief is that the nature of the computations underlying visual perception depend more on the computational problems that need to be solved than on the mechanism with which they are performed. Thus, the housefly is built for a few types of computations based on visual stimulus and achieves rapid process time. Human perceptions encompass a larger variety of probems (and in fact are culturally conditioned and to some extent are dependent on previous experience) and require correspondingly longer response times that are clearly dependent on the nature of the problem encountered.

Representation and algorithms are the second level of approach and understanding of this problem. With a computationally theoretic understanding of the nature of the visual problems to be solved, it is feasible to search for the manner in which the visual stimulus will be encoded, either in the brain or in the machine. This then becomes the study of representations. Given an understanding of the representations involved, it is then possible to consider useful and/or efficient algorithms for processing the information in its represented form. At this level of investigation, much effort has been devoted to the psychology of perception. It is clear that a visual scene may produce several types of representations, and the nature of the interpretative problem to be solved will engage appropriate mechanisms to process appropriate information. Some exploration of the complexity of the visual process is given by Wolfe [16] who shows that the process of creating a visual representation involves several interactive processes which he calls hidden visual processes. Wolfe describes several experiments designed to probe the response of various physiological systems involved in visual perception. One example is an experiment designed to explore edge detection as a function of brightness and color. This experiment shows that the common features we associate with vision are not isolated phenomenon. In this case, the conclusion is that edge detection is not a color-sensitive visual function: isoluminescent edges created by distinct colors are difficult to perceive. This research concludes that the full range of human visual senses or processes has not yet been discovered and that perception is a complex interaction amongst many functions of the human brain and nervous system. This is a concrete example, for the present discussion, of a central assertion that visual perception involves information, assumptions and constraints above and beyond the visual stimulus present in the scene.

In this context, Marr emphasizes the importance and nature of the concept of process. Process is a vague word whose meaning is open to the choice of the user; considered use of the work perhaps creates the awareness of the delicate balance of Lewis Carroll's Humpty Dumpty who stoutly insisted that words were defined by the meaning he chose to give them. Marr chooses to use the word process as a unifying description of the application of the methodology discussed. At the abstract level of computational theory, process is examined in terms of what it accomplishes and why. What the process does is associated with the rules of theory which describe the process; the why inherent in the use of the word relates to the constraints imposed by the real world on the nature of the desired results. At the representation and algorithm level of visual perception, process is the relation between the representation and the problem-solving choices. This encompassses the information derived from the computational theory and the understanding of *real-world constraints on interpretation and desired results*. At the hardware or implementation level, process becomes the understanding of the transformations of the representations by the algorithms into the desired results and the choice of appropriate tools for accomplishing these tasks.

At this point, before presenting some of the detailed analysis of the Julesz illusion, return to the word *process* as a device for relating this discussion of robotic vision to symmetry. The language used here is seductively similar to discussions of symmetry as a transformation process [3]; there is also a clear correspondence with the language in the earlier discussions of symmetry presented here. It is tempting to dismiss this similarity as a simple-minded analogy. However, Marr's three-level methodology is applicable to problems in symmetry in a manner which is more than coincidence. The computational theory level, with its emphasis on what and why is essential in deciding the existence and usefulness of the symmetry concept for any particular question. As we have seen, symmetry is often in the eyes of the beholder, depending on time frame, inertial coordinate base, or even the vision of artistic license which creates symmetry in ways that are previously not found. Representation, transformation, and associated algorithms (or rule of relation) are the essence of symmetry as a process of implicate action. And the hardware level is the choice of tools with which to express symmetry in a manner accessible to others. Equally important, this methodology explicitly acknowledges the incorporation of the constraints imposed by the real world at all three levels of understanding. These constraints represent implicit extra information that is not present in the direct information being processed.

At this point, a brief survey of Marr's analysis of the Julesz illusion provides a concrete example of the implementation of this methodology. This survey is not intended to be comprehensive but rather is an illustration of specific concepts involved in robotic vision, concepts that bear on the relationship between symmetry and machine perception. This analysis begins with a description of stereopsis based on the binocular images created by the difference in spatial position of two eyes. The spatial positions in these images are characterized by a displacement called disparity. Marr chooses to restrict this term to mean the angular discrepancy the displacement creates with respect to the visual field of each eye. The Julesz illusion provides a concrete visualization of the problems involved in determining the generalized stereo disparity and using this information to create the subjective perception of depth. At the level of computational theory, subjective analysis of these two images may be described in terms of matching identical points on the image as seen by each eye. In this particular example, some points bear a one-to-one spatial relationship between images while the translated segment contains others that do not. Even for the identical portions of the diagram, the 2-D projection of the two images does not contain sufficient information for a unique solution for the correct matching of the points as perceived by each eye. There is a fundamental ambiguity, known as the false-target problem, that permits multiple matching of points if perception depends only on simple ray-trace analysis of images. The Julesz illusion provides an accessible example for understanding the questions implicit in the false-target problem.

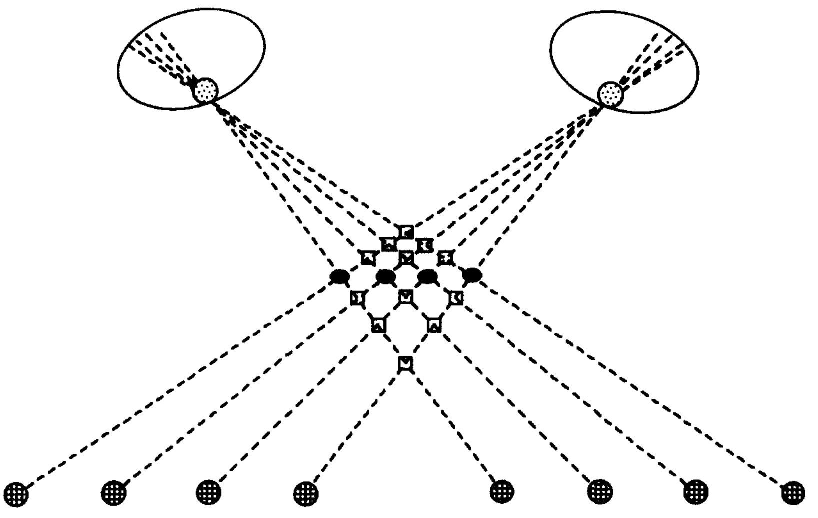


Fig. 6. Ray representation of false target problem central to stereo matching of left and right eye images: after the work of Marr.

Consider the diagram in Fig. 6. Given the images of the sets of dots that relate to each eye, correct perception requires unique identification of the dots. This represents the true correspondence of the images in the physical world. There are 16 possible matches in the figure, of which only four represent the existing situation being perceived. The unique solution is provided by additional constraints, originating in the physical world, that allow the correct matching of identical points of positions on an object. Marr describes two simple facts known from ordinary experience in the physical world that restrict the solution of this problem and allow a unique image.

Deceptively simple, these facts are that a given point on a physical surface does in fact have a unique spatial position at a given time (with respect to human perception) and that surfaces are usually smooth and continuous. These two facts are noticeably similar to the learned information postulated by Hoffman in his analysis of illusions [6]. Marr formulates these constraints in terms of matching: if two potentially corresponding elements can have arisen from the same physical location, they can match. Only two elements are in fact the correct match, and the continuity of surfaces means that the angular disparity between points varies smoothly almost everywhere. In terms of the Julesz illusion, these matching constraints may be translated into rules: *compatibility*—black dots can only match black dots: *uniqueness*—almost always a black dot from one image can match only one black dot in the other image; and *continuity*—the angular disparity of the matches varies smoothly over the image. Boundaries and experientially less probable alignment of objects may create exceptions, but most perceptual experience is subject to these constraints, that are *derived from the nature of the physical world.*

The applicability of this analysis results from the fact that most visual scenes can be conveniently divided into segments or tokens of various levels of complexity. Reduction of a scene into such elements produces a representation of the image commonly known as a primal sketch. Marr suggests that the creation of stereopsis implies the existence of a buffer memory process within the brain using these primitive tokens. He labels the contents of this buffer memory the 2 1/2-D sketch. Such an idea corresponds with many of the ideas presented that consider visual perception as the result of unconscious nonlogical mental processing. This implicit presumption of distinct elements is an integral part of the premise that visual perception is a computationally accessible process and is a significant aspect of the fundamental relationship between the symmetry concept and visual processing. (This is discussed at greater length in the next section.) At the computational level of theory, the matching constraints derived from experience in the physical world create the fundamental assumptions of stereopsis: given a scene containing sufficient detail and fulfilling these matching constraints for the elements of the images, the correspondence is then assumed by the mind to be unique and a correct perception. Marr presents convincing arguments that this analysis is sufficient for understanding stereopsis.

A detailed discussion of the proposed algorithms for solution of the Julesz illusion is too extensive to be adequately covered here. However, a brief description of two proposed methods of solution for this problem will illustrate the implementation of these considerations. Marr first proposes a cooperative algorithm based on calculating the probability of matching of two points as a function of disparity. By dividing the problem into elements representing correspondence between spatial positions perceived by each eye, the probabilty that a particular element represents a correct solution to the false target problem is formulated as an iterative relationship between elements modulated by probabilities based on the matching constraints discussed above. The scene is modeled as several planer arrays of elements, each plane corresponding to a different angular disparity such that the intersection element represents the matching between points in each image related by a given angular disparity. The probabilities are calculated based on the constraints which are expressed as directional relationships between the elements in each plane. The uniqueness constraint inhibits probabiliity of matching along horizontal and vertical directions in the planes (these directions representing the lines of sight for each eye). This is because a given position for one eye implies that there is only one position for the other eye that represents a correct match. The smoothness or continuity constraint amplifies the probability of correct match for diagonal directions as matching for one point implies that matching for another is likely to occur with small but distinct displacments in each line of sight. An iterative problem is formulated that distinguishes the matching of points in the Julesz images as a function of the disparity and correctly identifies the translation that has been performed. A second algorithm is postulated in terms of multiple matching processes, starting with rough correspondence and converging to matching of fine detail. In this formulation, a buffer image or representation of the scene is a central site for multiple types of processing and interaction, both with the sensory data which is incoming (eye movements are postulated to be important thus implying the processing of multiple images created for each scene) and for interaction with implicit and possibly quite complex mental transformations of the information (thus perhaps providing a link for such well-known effects as those of past experience or presumed spatial relationships). This second formulation emphasizes the complexity and large amount of additional information necessary for creating perception from the sensory information about the visual scene.

In summary, the problem has been analyzed in terms of purpose. For the Julesz illusion this is easily understood as one of establishing the relationship between corresponding dots in the two images. Given this purpose, the possible visual relationships between the dots has been explored in terms of the information available from the scene. The inherent ambiguity has been analyzed and proposed mechanisms for determining unique solutions have been developed. This has been done by the use of additional information obtained from constraints imposed by physical world experience. By using these constraints, possible quantification of the visual information has been proposed that differentiates the dots in a manner likely to be accessible to the visual system. Computational algorithms are then proposd and implemented that in fact identify the differentiation of the dots in the illusion in a manner that reproduces the structural features perceived by human stereopsis. For this discussion, this process of analysis and the implications of its extension to other visual perception is most significant. This presentation of the problems inherent in robotic vision reveals questions related to generalized human perception. It thus follows that questions about robotic vision are inexorably intertwined with the broad and timeless questions about the nature of human knowledge, language and understanding.

A notable example is the use of the word process discussed above. If vision and perception are approached from a broader view, such that the human brain/mind taken as a whole plays the role of "hardware" as envisioned by Marr, the intrinsic processing and transformation of visual information into conceptual perception has long been recognized. If one accepts the visual orientation of descriptive language, the historical recognition of this phenomenon is manifest. The word "imagery" is used in fundamental ways that far transcend any limitations imposed by confines of visual stimulus or recollection. With the dawn of analytical psychology, enormous effort went into attempts to create unifying theories or ideas about the nature of implicit information processing creates the ordinary understanding of *what is where*. The work of Kanizsa reiterates, in another manner, some of the important relational concepts mentioned above and sets the stage for creating some examples to illustrate the indigenous relationship between symmetry and visual perception. In the collection of essays constituting a volume entitled *Organization in Vision* [7], Kanizsa deals with numerous aspects of this general problem of understanding human visual perception. This volume provides access to the many years of literature on the subject and provides some helpful perspective on the question considered here.

In one chapter, Kanizsa discusses the conflicting information about the influence of past experience on present visual perception. There is no question that such learned response plays a role in the subjective experience of present visual information. It is well-known that indistinct objects (such as those seen from afar) are often perceived as familiar forms drawn from conditioned images inherent in the mind [8]. Earlier discussion describes the well-known illusion for which the mind "fills in the blanks", creating familiar images in the presence of only partial visual stimulus. Past experience is often so dominant that visual perception is created in conflict with the stimulus present; impossible objects such as the Penrose drawing, or many of the works of M. C. Escher draw on the fact that the 2-D projection presents ambiguous information (as discussed above) and illusionary effects may be created drawing on the instinctive experience of depth. It is clear that the basis artistic representation of perspective is created by everyday experience of the 3-D world. It has also been shown that this visual experience is culturally dependent; some social structures exist for which 3-D interpretation of 2-D scenes is not present and for whom the familiar depth illusions have no meaning [11].

To emphasize the role past experience plays in perception, consider the fact that the human mind possesses an astounding ability to recognize other individual human faces. A commentary by Garfield [17] reviews this question and reiterates several significant points. First, it is apparent that the human mind can recognize a familiar face in a crowd of several hundred strangers in less time that would be required for complete processing of the visual information present in the scene. This ability to recognize familiar faces also seems to last over long periods of time; twenty years may lapse and yet people still recognize a face familiar from that previous period. It is clear that faces are complex visual stimuli, often encountered, which are familiar, and trigger complex emotional responses; all of which may contribute to the special perceptual ability with which they are associated. This is without a doubt, an example of experience-influenced perception.

However, a recent unusual technological development suggests that the nature of such experiential influence is not obvious. As suggested in 1973 by Howard Chernoff (a statistician at Harvard University) a cartoon face may be used to represent a surprising amount of information [18] (see Fig. 7). By using shape characteristics of various facial features, it is easy to postulate up to 10 distinctive identifiers for such a cartoon face. Shapes of the mouth, eyes, nose, etc. create a distinct facial expression that is recognizable at a glance. Given 10 settings for each feature, i.e., 10 gradations between happy smile and an angry frown, 10 billion expressions are combinatorially distinct and subtle differences between these combinations are accessible. Given training, these differences can create emotional/intellectual meaning that is useful for conveying large amounts of information. It is interesting to conjecture the role that perceived symmetry plays in this process. It is well-known that human faces are not symmetrical [19] and, given the experiential basis for face recognition, it is reasonable to assume that this lack of symmetry is one component of the identification process. Given the mind's tendency to "fill in blanks" and to impose symmetry, it is perhaps the deviations from symmetry that provide the clues to individual identity and character. Clearly the imposition of bilateral symmetry on the data face representation problem is a limitation of allowed expressions. It is intriguing to speculate that the differences from symmetrical shape and orientation may be more readily accessible and identifiable than the complete image itself; this reduced amount of information could account for the exceptionally rapid recognition of a friend in the crowd.

To return to Kanizsa's discussion of experience-influenced perceptions, his primary focus is on the difficulty of identifying in a clear manner the exact nature of this influence. Spatial arrangement illusions, identification illusions, and shape/transparency illusions are presented to communicate that the effect of experience does not produce consistent perception derived from a visual scene. It is clear that a purely empirical, experiential based analysis is not sufficient to satisfactorily identify the nature and extent of the necessary extra information used for visual interpretation.

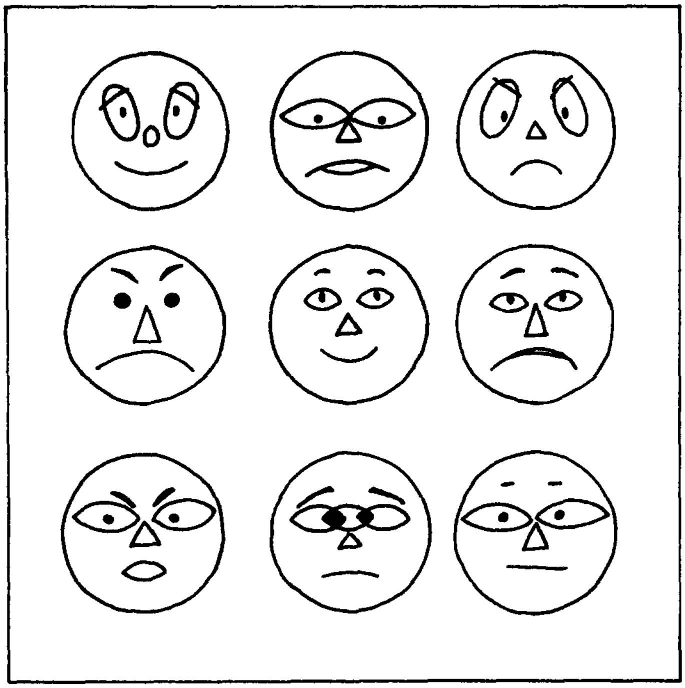


Fig. 7. Simple cartoon representations of possible data face constructions. Elimination of bilateral symmetry increases possible information content.

The first essay in Kanizsa's book concerns interpretation of information in ways that go beyond the extent of available information. He considers perception to be the process that selects, discards, analyzes and integrates sensory data; his emphasis on the word process is to avoid a separation between vision and thinking. In his first chapter the familiar illusions of impossible objects, ambiguous interpretations and image filling of blank space are encountered to show that careful analysis of such pathological scenes raises questions whose consideration can extend the breadth of human knowledge and understanding. Of interest here is the conjuction of questions about what can be thought and what can be seen (or perceived). A primary conclusion is that the eye reasons in its own fashion, i.e. that visual perception is an exercise of human thinking and analysis which may follow different rules than reasoning and logical processing. Kanizsa thus strongly suggests that sufficient understanding of visual perception to create robotic vision well reveal and use computational processes that are not immediately obvious or deducible from other problems or experience.

It is important to recognize that this discussion is basically an attempt to organize, in the terminology of technology, ideas and understanding that have been part of the human knowledge base for a large part of history and have been important in many cultures and civilizations. Visual thinking and process is different from logical reasoning and analytical organization of information. The distinctive role of visual art and esthetics in history and culture is common knowledge. A contemporary discussion of the role of visual thinking has been organized by Georg Kepes in a volume entitled *Education of Vision* [20] which is part of a series based on ideas related to vision + value. In his introduction, Kepes stresses the impact visual experience has upon the psychological function of the human mind and in this context presents vision as a continuous creative process. This discussion and terminology are directly related to the ideas presented above. The first article in the volume is "Visual thinking" by the psychologist Rudolph Arnheim [21]. This article elaborates many of the characteristic aspects of the mental processing that are inherent in seeing. It is not surprising, therefore, to find that the psychological analysis arrives at the same concepts of distinct unconscious computation and analysis that are necessary for an adequate methodology for the creation of robotic vision. In this vein, a volume entitled *The Psychology of*

Perspective and Renaissance Art by Michael Kubovy [22] provides an artistic and humanistic analysis of the stereopsis problem discussed in computational terms above.

It is worthwhile remembering the complexity of the visual system postulated by Wolfe [16] and the apparent cooperative function of several physiological systems that create sight. This directly reinforces Marr's insistence of the importance of computational theory. It also suggests that new information, understanding and knowledge will certainly appear as the development of successful machine vision continues. New technology will allow creation of specialized experiments in processing visual information. These experiments will involve testing of controlled parameters, information processing and analysis of results in ways that are not currently accessible in biological systems. It follows, as will be discussed later, that our understanding of symmetry will also be affected; new perceptions form the basis for new symmetries.

GENERAL IMPLICATIONS: RELATION TO SCIENTIFIC PROCESS

The previous discussions about symmetry and vision can be viewed as a natural extension of the considerable debate over the nature of scientific thought and process. As we have seen from the work of Russell, the questions of ambiguity and the problems of forming "correct" perceptions are inherent in discussion and determination of scientific information. Because symmetry is most often invoked explicitly in scientific considerations, some further discussion of these problems in the scientific context are given to describe general relationships that are the premise of this article. Most of this discussion is drawn from a collection of essays by Michael Polanyi entitled *Knowing and Being* [23]. However, it is not difficult to realize that the questions considered here have long been discussed in many contexts (see author's note). It thus follows that pursuit of the quest of robotic vision and fuller comprehension of the symmetry concept will entail broad implications about the generalized nature of human knowledge and understanding. It is often stated that much progress in science is predicated on advances in technology. It is contention of this article that technological progress creates fundamental changes in human thinking, including areas that are far broader than those primarily encompassed by scientific considerations.

The first arguments for consideration appear in an essay entitled "The unaccountable element in science." [23] Polyani's focus is on the singular contribution of an ordinary scientist dealing with an ordinary scientific problem. He considers the contributions to scientific thought made by personal judgements that are distinct and irreplaceable by explicit reasoning. Central to these ideas is the considered premise that raw experience is devoid of all meaning and is made intelligible only by the powers of perception. This premise uses much the same concept as proposed by Kanizsa [7] or Kepes [20] in attributing a distinct type of human thought process to visual organization and perception that is quite different from reasoning and logical deduction. Many such ideas are explored by Marr [12] of which the buffer memory proposal for the 2 1/2-D sketch is only one. Much of the discussion concerns the question of ascribing formalized significance to information, i.e. what are the considerations about the probability that a particular set of data has valid or useful significance. Polanyi cites a remark attributed to Enrico Fermi that a miracle is an event whose chances of occurrence are less than one in 10. Another such rule, ascribed to Sir Robert Fisher, rejects patterns with probabilities of less than one in 20 as illusionary. These considered definitions of miracles differ only by numerical ratio. The point of these remarks is that the boundary between chance and pattern is arbitrary.

Polanyi then describes mathematics as only a formalized link between an intuitive surmise of significance and an arbitrary, informal decision to accept or reject on some basis of computed probability. In this light, randomness becomes conceivable only in relation to potential order and a determination of either is the result of an informal act of personal interpretation. Thus progress in science is conjectured to be a distinctly personal effort guided by the gift of perceiving a problem that is not observed by others, sensing a personal direction in the midst of apparent randomness, and eventually creating a solution that is a surprise to others. Polanyi postulates a similarity between perception and scientific intuition. Both are governed by rules that are perhaps unknowable but are certainly individual and distinct from other forms of human thinking. He concludes that scientific knowledge is accepted on the premise of hidden truths and thought processes which then becomes the motivation for further investigation. In this manner Polanyi equates scientific process with the process of perception. Both processes are motivated by the need to understand the foundations of seeing and perception and both processes are founded on the premise that we can make sense of experience because it "hangs together in itself". This discussion bears a remarkable similarity to Marr's analysis of vision. For Marr [12], robotic vision is clearly founded on the premise that perception is computationally accessible. It can be created only by using implicit information not present in the visual scene. Constraints derived from real world experiences of uniqueness and continuity are necessary to create the sensation and confidence that what we are perceiving "hangs together in itself". This is also similar to the hidden processes of Wolfe [16] and learned laws of Hoffman [6]. This similarity of analysis and even the use of language is not coincidence; it is a manifestation of the broad scope of the developments in machine vision.

A further elaboration of Polanyi's ideas is found in the essay entitled "The logic of tacit inference" found in the same volume [23]. Having concluded that understanding logical reasoning and deduction is not a sufficient basis for understanding the scientific process, Polanyi then looks for the area of human experience from which to draw information that will contribute to his analysis. He wants to find some human logic by which tacit or assumed mental processing (perhaps unconscious or unknowable) can achieve and uphold valid conclusions. Again, the example of perception becomes central to the discussion. He maintains that the capacity of science to perceive new and unique patterns differs from ordinary perception only because it has tools and training to integrate shapes that are not readily handled by ordinary perception. Trained perception is asserted to be the basis for all descriptive sciences. He concludes that there is no justification for separate approaches to scientific explanation, scientific discovery, learning and meaning; they are unified aspects of a general process of perception or understanding.

It thus seems reasonable to consider Marr's work (and in fact the whole effort directed towards robotic vision) as examples of generalized attempts to create representations and understanding of human thought and reasoning. It is significant that Marr's analysis of robotic vision encounters difficulties in creating perception from ambiguous information in a practical engineering context that is similar to those Polanyi encounters and describes in his philosophical endeavor for creating descriptions and understanding of scientific thought and process. It follows that the similarity of ideas and language result from treating inherently simlar questions and problems. From this, symmetry as a trained perception created and used for particular applications, is another example of such a process.

To this point, this essay is concerned primarily with the construction of ideas to show the similarity of the fundamental problems encountered in creating robotic vision and in the use and analysis of the symmetry concept. There appears to be a profound basis for this similarity in the phenomenon of the physical world. Polanyi boldly asserts the lack of fundamental meaning in raw experience; Marr deals with machine perception in terms of creating coherent pattern based on ambiguous information. From the discussions of Prigogine [24] concerning trajectory analysis and thermodynamics, a concept of a fundamental physical reality that corresponds to these ideas can be derived. From the extensive and profound presentation by Prigogine in the book *Order Out of Chaos* [24], the ideas relevant to this essay are summarized:

(i) The principles of thermodynamics necessitate that any given pattern or order will evolve in phase space in the course of forward time into chaotic equilibrium.
(ii) The second law of thermodynamics acts as a selection principle so that only those intial conditions which will lead to this chaotic equilibrium may occur for a given system.
(iii) It follows that any apparent pattern or order contains within it the same degree of chaos or randomness, in terms of its trajectory through phase space as is found in the final equilibrium.

It thus appears that ambiguity and disorder are inherent in the facets of the physical world that create the stimulus for our perceptions. It is resonable to believe that this uncertainty is rooted in all our attempts to deal with the environment in which we are inextricably immersed. Polanyi himself suggests that a Laplacean mind that computes future trajectories from present topography is merely transforming meaningless information into other meaningless information and not adding to the advancement of knowledge.

To conclude this section, it is interesting to note that Polanyi rejects a cybernetic interpretation of thought and behaviorism. This is counter to Marr's assumption that perception is computationally accessible. It appears likely that the debate between these two beliefs will focus not on the information present for mental or computational processing but rather on the nature and extent of the rules or extra information necessary for creating useful interpretations or patterns derived from the experiential information.

CONCLUSION: NEW WINE FROM NEW WINESKINS

The conclusion of the essay is expressed by the title "Mind's eye". The fundamental similarities between problems in robotic vision and those in symmetry arise from functioning of the central core of human perception, which is perhaps an almost trivial tautology. However, the many levels of understanding inherent and necessary for an adequate analysis of problems in either discipline give rise to rather more profound observations concerning future developments in both areas.

A central theme of this essay is the fundamental role of ambiguity inherent in the problems addressed by each area of research. In some sense, even the elementary mathematical analysis of textbook symmetry is a language for dealing with ambiguity. Transformation-related equivalence organizes vast amounts of perception into a concise and manageable representation. With the increasing sophistication of theories and scope associated with the symmetry concept, new computational technologies for processing information present in a visual scene and/or necessary for representing such information in an accessible manner will emerge. Conversely, new technologies for machine vision will suggest new manners of organizing information and will fuel development of complex ideas in symmetry.

It is also clear that technological developments in pursuit of robotic vision will generate new types of information. The developments of image-processing methods have created new information; false-color representation, image enhancement and the associated mathematics of pixel processing, and the basic efforts to identify fundamental elements of visual scenes already have created new types of information for which the technology associated with symmetry has been essential. The exponential growth of computational power, which has kept robotic vision as a viable carrot in the competition for useful applications, has simultaneously created the exploration of unforeseeable complexities in symmetry. It is also clear that the search for generalized robotic vision, driven, in Marr's terms, by the need to create a machine that knows *what is where*, will also spin off special-purpose technology designed to see *what* man cannot see and, certainly, *where* man cannot see. The type of perceptions created by such developments are bounded only by the limits of the imagination. It follows that new symmetries, new equivalences, and transformations, will result derived from specialized definitions of purpose.

With the acceptance of a boundless human imagination and the limitless vision of the "Mind's eye", the concluding assertion here is that the technological developments will irrevocably affect our understanding of human perception, knowledge and understanding. These timeless wonders are still at the center of revolutions in human ideas because the very assumptions and hidden processes behind them are subject to innovative challenge and examination.

Author's Note

At the request of the editor, I give here a few notes about the title of this article. Many readers will have seen this phrase or idea in many different contexts. The idea that human vision creates a perception of the world that is distinct from some more fundamental physical reality is found throughout recorded history. Philosophy and religion have been the natural areas of human culture for the propagation and survival of the ideas about this question which were part of ancient cultures. In a survey of the philosophies of India, Heinrich Zimmer [25] traces an origin of this concept to the Vedic religion of the Aryan culture which began in the frame of 2000–1000 B.C. He posits two fundamental lines of thinking which clearly address this concern; the first was the inquiry into the essential nature of the physical reality from which vision derives such changing perceptions; the second was into the basic functioning of the human self which controlled the essential creative processes of perception. A sense of this ancient understanding of this dilemma may be found in the following quote from the Brihadaranyaka Upanishad [26]

> "He is the unseen seer, the unheard hearer, the unthought thinker, the ununderstood understander. No other seer than He is there, no other hearer than He, no other thinker than He, no other understander than He: He is the Self within you, the Inner Controller, the Immortal. What is other than He suffers."

The Platonic formulation of our perceptions being derived from images which are shadows of a true reality cast on the wall of the cave of our existence is a similar root in Western culture. The mystical pursuit of direct perception of reality is a transcultural phenomenon based on this recognition that the true eye lies within the mind.

To conclude the note, an extensive survey of contemporary exploration of this type of problem, as well as many others raised by the concept of artificial intelligence may be found in *The Mind's I* [27].

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